A Scalable Limited Feedback Design for Network MIMO using Per-Cell Product Codebook

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***Abstract*—In network MIMO systems, channel state information is required at the transmitter side to multiplex users in the spatial domain. Since perfect channel knowledge is difficult to obtain in practice, *limited feedback* is a widely accepted solution. The *dynamic number of cooperating BSs* and *heterogeneous path loss effects* of network MIMO systems pose new challenges on limited feedback design. In this paper, we propose a scalable limited feedback design for network MIMO systems with multiple base stations, multiple users and multiple data streams for each user. We propose a *limited feedback framework using per-cell product codebooks*, along with a *low-complexity feedback indices selection algorithm*. We show that the proposed per-cell product codebook limited feedback design can asymptotically achieve the same performance as the joint-cell codebook approach. We also derive an asymptotic *per-user throughput loss* due to limited feedback with per-cell product codebooks. Based on that, we show that when the number of per-user feedback-bits B_k is $\mathcal{O}\big(Nn_Tn_R\log_2(\rho g_k^{sum})\big)$, the system operates in the *noise-limited* regime in which the per-user throughput is $\mathcal{O}\left(n_R\log_2\left(\frac{n_R\rho g_k^{sum}}{Nn_T}\right)\right)$. On the other hand, when the number of per-user feedback-bits B_k does not scale with the *system SNR* ρ, the system operates in the *interference-limited* regime where the per-user throughput is $\mathcal{O}\left(\frac{n_RB_k}{(Nn_T)^2}\right)$. Numerical results show that the proposed design is very flexible to accommodate dynamic number of cooperating BSs and achieves much better performance compared with other baselines (such as the Givens rotation approach).**

***Index Terms*—Network MIMO, Limited Feedback, Per-cell Product Codebook, SDMA, Performance Analysis**

I. Introduction

Network MIMO (multiple-input multiple-output) is considered as a core technology for the next generation wireless systems. The key idea of network MIMO is to employ base station (BS) cooperation among the neighboring cells for joint signal transmission in downlink direction and/or joint signal detection in uplink direction [1]–[4]. In network MIMO systems, the undesirable inter-cell interference (ICI) can be transformed into useful signals via collaborative transmission among multiple adjacent BSs. Therefore, the network MIMO solution could effectively leverage the advantage of MIMO communications.

Similar to single-cell multiuser MIMO (MU-MIMO) communications, knowledge of channel state information (CSI) is critical for efficient spatial multiplexing of mobiles in network MIMO systems. Space division multiple access (SDMA) for single-cell MU-MIMO has been studied in lots of literatures [5]–[11]. In [5], [6], perfect knowledge of CSI at the transmitter is assumed to eliminate cochannel interference (CCI) among the users engaged in SDMA. However, perfect CSI is difficult to obtain at the transmitter side in practice and there are lots of literatures discussing SDMA with limited CSI feedback in single-cell MU-MIMO systems [7]–[13]. Recently, the authors of [14], [15] have extended the single-cell limited feedback designs to network MIMO systems by treating the cooperating BSs as a composite MIMO transmitter (i.e., one super BS), and this refers to the *joint-cell codebook* approach. While the existing work [14], [15] provide some preliminary solutions for CSI feedback in network MIMO systems, there are still a number of important issues to be addressed.

- **Dynamic Number of Cooperating BSs:** One important difference between single-cell MIMO and network MIMO systems is that the number of cooperating BSs in the latter case is dynamic, depending on location of the mobiles. As a result, the total number of bits for CSI feedback[1] as well as the dimension of the CSI matrix seen by a user are dynamic. The conventional limited feedback designs are all designed for fixed number of transmit antennas and cannot be directly applied to network MIMO systems due to the lack of flexibility. In other words, it is very important to have flexibility incorporated in the codebook-based limited feedback schemes in network MIMO systems, so that the same codebook can be used to quantize the CSI matrix seen by a user regardless of the number of cooperating BSs. This poses a new design criteria for limited feedback mechanisms in network MIMO systems.
- **Heterogeneous Path Loss Effects[2]:** In network MIMO systems, it is quite common to have non-uniform path losses between a mobile station (MS) and the cooperating BSs. Hence, the conventional Grassmannian codebooks [16]–[18], which is designed to match the CSI matrix with i.i.d. entries, fail to match the actual statistics of the *aggregate CSI matrix* seen by a user due to the heterogeneous path loss effects. In addition, the path losses geometry seen by one MS are dynamic and cannot be incorporated into the offline codebook design procedures.
- **Performance Analysis:** The analytical results of the

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[1] In LTE-Advanced systems, the total number of bits for limited feedback scales linearly with the number of active BSs.

[2] "Heterogeneous path loss effects" refers to the different path losses from the N cooperating BSs to one MS.

single cell SDMA scheme with limited feedback has been considered extensively in the literatures [8]–[11]. However, these results cannot be applied to the multi-cell scenario with limited feedback capturing the dynamic number of cooperating BSs and the heterogeneous path loss effects.

One conventional limited feedback design, namely the Givens rotation [19]–[21], could potentially address the above challenges. Using Givens rotation, a unitary matrix (channel direction) is decomposed into products of Givens matrices. Each Givens matrix contains two Givens parameters, which are quantized using *scalar* quantizer and fed back to the BSs. As a result, it offers the flexibility because when the number of cooperating BSs changes, the number of Givens matrices also changes accordingly. Hence, the same scalar quantizer can be used to quantize unitary matrices of time-varying dimensions. However, the issue of Givens rotation approach is the poor feedback efficiency due to scalar (or two-dimensional vector) quantization. In this paper, we shall propose a novel scalable limited feedback mechanism using *per-cell product codebooks*[3] to address the dynamic MIMO configurations and heterogeneous path loss effects, along with a low-complexity realtime feedback indices selection algorithm. In the proposed feedback scheme, the *product codebook* (defined in Section III-B) that is used for CSI feedback in the network MIMO systems is simply the *Cartesian product* of N per-cell product codebooks, with N denoting the number of cooperating BSs. *Cartesian product* operation allows for a single per-cell product codebook to be used irrespective of the number of cooperating BSs. We shall show that the proposed per-cell product codebook based limited feedback mechanism can asymptotically achieve the same performance as the joint-cell codebook approach. We derive an asymptotic *per-user throughput loss* due to limited feedback with per-cell product codebooks. Based on the results, we show that when the number of per-user feedback-bits B_k is $\mathcal{O}\big(Nn_Tn_R\log_2(\rho g_k^{sum})\big)$, the system operates in a noise-limited regime with per-user throughput scaling as $\mathcal{O}\left(n_R\log_2\big(\frac{n_R\rho g_k^{sum}}{Nn_T}\big)\right)$. On the other hand, when the number of per-user feedback-bits B_k does not scale with the *system SNR* ρ, the system operates in an interference limited regime with per user throughput scaling as $\mathcal{O}\left(\frac{n_RB_k}{(Nn_T)^2}\right)$. Numerical results show while the proposed scheme is flexible to accommodate dynamic number of co-operating BS, it has significant performance gains over the reference baselines (e.g. Givens rotation approach).

The rest of this paper is organized as follows. We introduce the network MIMO system model and codebook-based CSI feedback model in Section II. The proposed per-cell product codebook based limited feedback framework is introduced in Section III. Asymptotic performance analysis of the proposed scheme is elaborated in Section IV. We present the numerical results along with discussions in Section V. Finally, we summarize the main results in Section VI.

[3]"Per-cell product codebook" refers to the codebook that is designed with the single BS antenna configuration; while "joint-cell codebook" refers to the codebook that is designed with the aggregate antenna configuration of the N cooperating BSs. In other words, in joint-cell codebook design, the N cooperating BSs are treated as one aggregate BS, which is called super-BS in the paper.

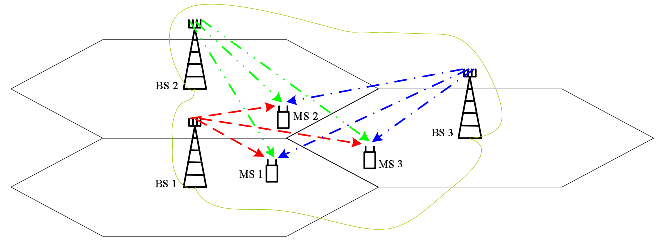

Fig. 1. An sample network MIMO system with $(n_T, N, n_R, K) = (4, 3, 2, 6)$. The 3 BSs collaboratively serve the 3 MSs via multi-cell SDMA

Notations: Matrices and vectors are denoted with boldface uppercase and lowercase letters, respectively; $\mathbf{A}^\dagger$ and $\mathrm{tr}\{\mathbf{A}\}$ denote the conjugate transpose and trace of matrix $\mathbf{A}$, respectively; $\mathbf{I}_L$ represents the $L\times L$ identity matrix; $\mathbb{CN}(\mu,\sigma^2)$ denotes the circularly symmetric complex Gaussian distribution, with mean μ and variance σ^2; $\mathbb{C}$ and $\mathbb{R}_+$ denote the set of complex numbers and positive real numbers, respectively.

II. System Model

A. Network MIMO Channel Model

Consider a network MIMO system (n_T, N, n_R, K) with N BSs serving K active MSs in the downlink direction, as shown in Fig. 1. The N BSs are inter-connected via high-speed backhauls, and collaboratively serve the K MSs through the standard SDMA scheme. Without loss of generality, we assume that each BS has n_T antennas and each MS has n_R antennas[4]. Assume *limited feedback based block diagonalization* (LF-BD) [7], [10] is employed for SDMA transmission in the network MIMO system, the downlink signal model can be written as,

$$\mathbf{y}_k=\mathbf{H}_k\widehat{\mathbf{W}}_k\sqrt{\mathbf{P}_k}\mathbf{d}_k+\underbrace{\mathbf{H}_k\Big(\sum_{j=1,j\neq k}^{K}\widehat{\mathbf{W}}_j\sqrt{\mathbf{P}_j}\mathbf{d}_j\Big)}_{\text{residual CCI}}+\mathbf{z}_k, \quad (1)$$

where $\mathbf{y}_k\in\mathbb{C}^{n_R\times 1}$ is the received signal vector at the k^{th} MS; $\mathbf{H}_k\in\mathbb{C}^{n_R\times Nn_T}$ denotes the *aggregate CSI matrix* of the k^{th} MS; $\widehat{\mathbf{W}}_k\in\mathbb{C}^{Nn_T\times n_R}$ is the precoder for the k^{th} MS, with $\widehat{\mathbf{W}}_k^\dagger\widehat{\mathbf{W}}_k=\mathbf{I}_{n_R}$; $\mathbf{P}_k=\frac{NP_{max}}{Kn_R}\mathbf{I}_{n_R}$ is the power allocation matrix for the k^{th} MS, with P_{max} representing the maximal transmit power of one BS; $\mathbf{d}_k\in\mathbb{C}^{n_R\times 1}$ is the transmitted symbol vector intended for the k^{th} MS, satisfying $\mathbb{E}\left\{\mathbf{d}_k\mathbf{d}_k^\dagger\right\}=\mathbf{I}_{n_R}$; $\mathbf{z}_k\in\mathbb{C}^{n_R\times 1}$ denotes the noise vector, with i.i.d. $\mathbb{CN}(0,\sigma^2)$ entries, i.e., $\mathbb{E}\left\{\mathbf{z}_k\mathbf{z}_k^\dagger\right\}=\sigma^2\mathbf{I}_{n_R}$. In this paper, it is assumed that $Kn_R\leq Nn_T$.

For precoder design, LF-BD imposes the following conditions on $\widehat{\mathbf{W}}_k$ to eliminate CCI at the transmitter side [7], [10],

$$\widehat{\mathbf{H}}_k\widehat{\mathbf{W}}_j=\mathbf{0}_{n_R\times L},\ \forall\ k\neq j;\ k,j=1,2,\cdots,K, \quad (2)$$

[4]Although we assume that the cooperating BSs and active MSs have homogeneous antenna configurations (i.e., every BS has n_T transmit antennas, and every MS has n_R receive antennas.), the proposed per-cell limited feedback mechanism can be directly extended to the cases with heterogeneous antenna configurations (i.e., different BSs and/or different MSs have different number of antennas.).

where $\widehat{\mathbf{H}}_k$ is the quantized aggregated CSI matrix of the k^{th} MS (see equation (12)).

In the network MIMO system, the *aggregate CSI matrix* $\mathbf{H}_k$ seen by the k^{th} MS can be partitioned into N submatrices, i.e.,

$$\mathbf{H}_k = [\mathbf{H}_{k,1} \;\; \mathbf{H}_{k,2} \;\; \cdots \;\; \mathbf{H}_{k,N}], \forall \; k = 1, 2, \cdots, K, \quad (3)$$

where $\mathbf{H}_{k,n} \in \mathbb{C}^{n_R \times n_T}$ denotes the CSI matrix between the n^{th} BS and the k^{th} MS, which is commonly modeled as [22],

$$\mathbf{H}_{k,n} = \sqrt{g_{k,n} s_{k,n}} \mathbf{H}_{k,n}^{(w)}, \forall k = 1, 2, \cdots, K; \; n = 1, 2, \cdots, N, \quad (4)$$

where $g_{k,n} \in \mathbb{R}_+$ denotes the path loss from the n^{th} BS to k^{th} MS; $s_{k,n} \in \mathbb{R}_+$ denotes the lognormal shadowing component; $\mathbf{H}_{k,n}^{(w)} \in \mathbb{C}^{n_R \times n_T}$ denotes a random matrix with i.i.d. $\mathbb{CN}(0, 1)$ entries, i.e., the Rayleigh fading component.

Moreover, the *aggregate CSI matrix* $\mathbf{H}_k$ of the k^{th} MS can be factorized as,

$$\mathbf{H}_k = \mathbf{H}_k^{(w)} \mathbf{G}_k, \; \forall \; k = 1, 2, \cdots, K, \quad (5)$$

where $\mathbf{H}_k^{(w)} \in \mathbb{C}^{n_R \times N n_T}$ denotes a random matrix with i.i.d. $\mathbb{CN}(0, 1)$ entries; $\mathbf{G}_k \in \mathbb{R}_+^{N n_T \times N n_T}$ represents the large-scale fading component, which is given by,

$$\mathbf{G}_k = diag\Big(\big[\sqrt{g_{k,1} s_{k,1}} \mathbf{1}_{n_T} \;\; \sqrt{g_{k,2} s_{k,2}} \mathbf{1}_{n_T} \;\; \cdots \;\; \sqrt{g_{k,N} s_{k,N}} \mathbf{1}_{n_T} \big] \Big), \quad (6)$$

where $\mathbf{1}_{n_T}$ equals to $[1\,1\,\cdots\,1] \in \mathbb{R}_+^{1 \times n_T}$, and $diag(\mathbf{a})$ denotes a diagonal matrix with diagonal entries given by the elements of vector $\mathbf{a}$.

B. Codebook-based Limited Feedback Model

For downlink transmission with the LF-BD scheme, the essential information (i.e., users channel subspaces) required at the BSs side can be obtained through the *codebook-based CSI feedback* scheme. In a general codebook-based CSI feedback framework, the design metric[5] for minimizing the residual CCI is *chordal distance* [7], [10], which is defined as,

$$d_c(\mathbf{V}_1, \; \mathbf{V}_2) = \frac{1}{\sqrt{2}} \left\| \mathbf{V}_1 \mathbf{V}_1^\dagger - \mathbf{V}_2 \mathbf{V}_2^\dagger \right\|_F, \quad (7)$$

where $\mathbf{V}_1, \; \mathbf{V}_2 \in \mathbb{C}^{N n_T \times n_R}$ are two orthonormal matrices[6]; and $\|\mathbf{A}\|_F$ denotes the Frobenius norm of matrix $\mathbf{A}$.

Without loss of generality, we assume the BS and MS share the same CSI quantization codebook, denoted by φ. The J^{th} codeword in the codebook φ is given by $\mathbf{V}_J$ and the cardinality of the codebook φ is 2^B, with B representing the number of feedback-bits. In the conventional single-cell MIMO communications, the codeword index J^* reported by the MS is generated through the minimization of the *chordal distance* between the quantization source $\mathbf{V}$ and the codedwords in the entire codebook φ. Mathematically,

$$J^* = \underset{\mathbf{V}_J \in \varphi}{\operatorname{argmin}} \; d_c(\mathbf{V}_J, \mathbf{V}). \quad (8)$$

The following assumptions are made through the rest of this paper. Firstly, all the MSs have perfect channel state information between the cooperative BSs and themselves. Secondly, the large-scale fading component $\mathbf{G}_k$ is known at all the BSs and MSs within the network. Thirdly, each MS shall feed back the CSI knowledge to the cooperative BSs through zero-delay error-free feedback links. In addition, flat block-fading MIMO channel is assumed.

III. Scalable Limited Feedback Mechanism based on Per-Cell Product Codebook

In the joint-cell codebook based CSI feedback schemes [14], [15], the cooperating BSs are treated as one composite BS (super BS), and the codebooks are designed for the *aggregate CSI matrix* $\mathbf{H}_k$ in a standard way. Moreover, authors of [15] has also considered the heterogeneous path loss effects and the effects of increasing cluster size with a variable number of cooperating BSs. While the joint-cell codebook approaches provide some preliminary solutions for network MIMO systems, two important issues, namely the dynamic number of cooperating BSs and the heterogeneous path loss effects, are ignored. In this section, we shall introduce a *scalable limited feedback design based on per-cell product codebook* to accommodate the dynamic MIMO configurations and deal with the heterogeneous path loss effects. Moreover, we shall formulate the realtime feedback indices determination (at the mobile) as a *combinatorial search* problem, and propose a low-complexity solution.

A. Scalable Per-cell Product Codebook based Limited Feedback Design

In order to accommodate the dynamics of cooperating BSs and deal with the heterogeneous path loss effects, we propose to quantize $span\left\{\mathbf{H}_k^{(w)}\right\}$ with per-cell product codebooks, rather than quantizing $span\{\mathbf{H}_k\}$ directly, where $span\{\mathbf{A}\}$ denotes the *row space* of matrix $\mathbf{A}$. The proposed per-cell product codebook based CSI quantization procedure involves both MS side processing and BS side processing, i.e., codewords indices generation at the MS side and quantized aggregate CSI matrix reconstruction at the BS side. Denote the N per-cell product codebooks as $\varphi_1, \varphi_2, \cdots, \varphi_N$, where $\varphi_n = \left\{\mathbf{V} \middle| \mathbf{V} \in \mathbb{C}^{n_T \times n_R}, \mathbf{V}^\dagger \mathbf{V} = \mathbf{I}_{n_R}\right\}$, with cardinality $2^{B_{k,n}}$, and the total number of per-user feedback-bits B_k is given by $B_k = \sum_{n=1}^{N} B_{k,n}$. Note that the N per-cell product codebooks could be identical. The proposed scalable limited feedback processing at the MS and the BS sides is summarized below:

Feedback Indices Generation (MS side): take the k^{th} MS as an example. The feedback indices generation is a mapping from the normalized aggregated CSI matrix $\mathbf{H}_k^{(w)}$ to the feedback indices $\left\{J_{k,1}^*, \; J_{k,2}^*, \; \cdots, \; J_{k,N}^*\right\}$ corresponding to the N per-cell product codebooks $\varphi_1, \varphi_2, \cdots, \varphi_N$.

[5]For limited feedback based SDMA transmission, a widely adopted design metric for CSI feedback schemes is *minimizing the inter-user interference*, rather than maximizing the desired signal power [7], [10].

[6]In this paper, an orthonormal matrix refers to a matrix with *orthonormal columns*.

- **Normalization**: To handle the heterogeneous path loss effects, we first normalize the aggregate CSI matrix $\mathbf{H}_k$ by the large-scale fading component $\mathbf{G}_k$, i.e.,

$$\mathbf{H}_k^{(w)} = \mathbf{H}_k \mathbf{G}_k^{-1}. \tag{9}$$

- **Decomposition**: Apply the standard *singular value decomposition* (SVD) [23] to the normalized CSI matrix $\mathbf{H}_k^{(w)}$, we have,

$$\begin{aligned}\mathbf{H}_k^{(w)} &= \mathbf{U}_k^{(w)} \left[\mathbf{S}_k^{(w)} \ \mathbf{0}_{n_R \times N n_T - n_R}\right] \left[\mathbf{V}_k^{(w)} \ \tilde{\mathbf{V}}_k^{(w)}\right]^\dagger \\ &= \mathbf{U}_k^{(w)} \mathbf{S}_k^{(w)} \left[\mathbf{V}_k^{(w)}\right]^\dagger. \end{aligned} \tag{10}$$

- **Realtime Feedback Indices Selection:** $\mathbf{V}_k^{(w)}$ is then mapped into N per-cell product codebooks with indices $\left\{J_{k,1}^*,\ J_{k,2}^*,\ \cdots,\ J_{k,N}^*\right\}$, and these codewords indices are fed back to the BSs via an error-free feedback link. The realtime indices generation is formulated as a combinatorial optimization in Section III-B.

Aggregate CSI Matrix Reconstruction and SDMA Precoder Computation (BS side): After collecting all the indices from the K MSs, the BSs try to reconstruct the CSI matrices and compute the SDMA precoder from the N codeword indices $\left\{J_{k,1}^*,\ J_{k,2}^*,\ \cdots,\ J_{k,N}^*\right\}$.

- **Reconstruction**: The indices $\left\{J_{k,1}^*, J_{k,2}^*, \cdots, J_{k,N}^*\right\}$ from the k^{th} MS are used to construct a quantized version of $\mathbf{V}_k^{(w)}$ (denoted as $\widehat{\mathbf{V}}_k^{(w)}$), i.e.,

$$\widehat{\mathbf{V}}_k^{(w)} = \frac{1}{\sqrt{N}} \left[\mathbf{V}_{J_{k,1}^*}^\dagger \ \mathbf{V}_{J_{k,2}^*}^\dagger \ \cdots \ \mathbf{V}_{J_{k,N}^*}^\dagger\right]^\dagger. \tag{11}$$

- **Denormalization**: $\widehat{\mathbf{V}}_k^{(w)}$ is used to construct the quantized aggregate CSI matrix $\widehat{\mathbf{H}}_k$ of the k^{th} MS, which is given by,

$$\widehat{\mathbf{H}}_k = \left[\widehat{\mathbf{V}}_k^{(w)}\right]^\dagger \mathbf{G}_k, \ \forall\ k = 1, 2, \cdots, K. \tag{12}$$

- **SDMA Precoder Computation**: Apply the standard SVD to the quantized interference channel $\widehat{\mathbf{H}}_{-k}$ seen by the k^{th} MS, we have,

$$\widehat{\mathbf{H}}_{-k} = \widehat{\mathbf{U}}_{-k} \widehat{\mathbf{S}}_{-k} \left[\widehat{\mathbf{V}}_{-k} \ \ \widehat{\tilde{\mathbf{V}}}_{-k}\right]^\dagger, \tag{13}$$

where $\widehat{\mathbf{H}}_{-k}$ is given by,

$$\hat{\mathbf{H}}_{-k} = \left[\hat{\mathbf{H}}_1^\dagger \ \hat{\mathbf{H}}_2^\dagger \cdots \hat{\mathbf{H}}_{k-1}^\dagger \ \ \hat{\mathbf{H}}_{k+1}^\dagger \cdots \hat{\mathbf{H}}_K^\dagger\right]^\dagger. \tag{14}$$

The SVD operation (13) generates an orthonormal basis of the *right null space* of $\widehat{\mathbf{H}}_{-k}$, i.e., $\widehat{\tilde{\mathbf{V}}}_{-k} \in \mathbb{C}^{N n_T \times n_R}$. We can set $\widehat{\mathbf{W}}_k = \widehat{\tilde{\mathbf{V}}}_{-k}$, which satisfies equation (2).

Remark 1 (Ways of Sending back Feedback Indices): When the k^{th} MS sends back the feedback indices $\left\{J_{k,1}^*,\ J_{k,2}^*, \cdots, J_{k,N}^*\right\}$, it could the feedback index $J_{k,n}^*$ the n^{th} BS, $\forall n = 1, 2, \cdots, N$; or it could send all the feedback indices $\left\{J_{k,1}^*,\ J_{k,2}^*,\ \cdots,\ J_{k,N}^*\right\}$ to its nearest BS; or it could simply broadcast all the feedback indices $\left\{J_{k,1}^*,\ J_{k,2}^*,\ \cdots,\ J_{k,N}^*\right\}$, which will be received by all the N cooperating BSs thanks to the broadcast nature of the wireless media.

Remark 2 (Generalization of Common Cooperating BSs): Without loss of generality, we have assumed that the K MSs have the same set of cooperating BSs. The above per-cell product codebook limited feedback framework can also be applied directly to the case where each MS has a different active cooperating BSs set. For example, suppose MS-1 has BS-1 and BS-2 as its active set and MS-2 has BS-2 and BS-3 as its active set. This can be accommodated by our framework by considering a common active set of BS-1, BS-2, BS-3 for both MS-1 and MS-2 and setting $g_{1,3} = g_{2,1} = 0$.

As a summary, the proposed per-cell product codebook based limited feedback mechanism has the following advantages.

- The proposed scheme relies on per-cell product codebooks, which are designed offline based on single-BS MIMO configurations. The proposed scheme is scalable w.r.t. any number of cooperating BSs.
- Standard precoder codebooks (such as the Grassmannian codebook, Lloyd's codebook, etc.) can be used in the proposed framework because the heterogeneous path loss issue is handled realtime in equation (9) and (12).
- We could further exploit the special structure of N per-cell product codebooks in the proposed framework to derive a low complexity feedback indices selection algorithm.

B. Problem Formulation

The feedback indices generation at the MS side is nontrivial, since it involves *combinatorial search* over the N per-cell product codebooks. In the following, we shall first define the *aggregate codeword* for the N per-cell product codebooks and then formulate the *feedback indices generation problem* as a *chordal distance minimization problem* between the aggregate-codeword and the quantization source.

Definition 1 (aggregate-codeword): Let $\mathbf{V}_{J_{k,n}} \in \mathbb{C}^{n_T \times n_R}$ denote the $J_{k,n}$-th codeword in codebook φ_n, an *aggregate-codeword* $\bar{\mathbf{V}}\left(J_{k,1}, J_{k,2}, \cdots, J_{k,N}\right)$ is defined to be,

$$\bar{\mathbf{V}}\left(J_{k,1}, J_{k,2}, \cdots, J_{k,N}\right) = \frac{1}{\sqrt{N}} \left[\mathbf{V}_{J_{k,1}}^\dagger \ \mathbf{V}_{J_{k,2}}^\dagger \ \cdots \ \mathbf{V}_{J_{k,N}}^\dagger\right]^\dagger, \tag{15}$$

where $\mathbf{V}_{J_{k,n}} \in \varphi_n$, $\forall n = 1, 2, \cdots, N$. ■

By the definition of the aggregate-codeword, the feedback indices at k^{th} MS side can be determined through the following optimization problem.

Problem 1 (Optimal Feedback Indices Generation): Finding out N codewords indices, that are denoted as $\left\{J_{k,1}^*,\ J_{k,2}^*,\ \cdots,\ J_{k,N}^*\right\}$, in the N per-cell product codebooks $\varphi_1, \varphi_2, \cdots, \varphi_N$ respectively, such that the *chordal distance* between the *aggregate-codeword* $\bar{\mathbf{V}}\left(J_{k,1}, J_{k,2}, \cdots, J_{k,N}\right)$ and the quantization source $\mathbf{V}_k^{(w)}$ is minimized. Mathematically, the feedback indices generation problem can be modeled as the following optimization problem,

$$\min_{J_{k,1}, J_{k,2}, \cdots, J_{k,N}} d_c\left(\bar{\mathbf{V}}\left(J_{k,1}, J_{k,2}, \cdots, J_{k,N}\right), \mathbf{V}_k^{(w)}\right) \tag{16}$$

$$\text{subject to} \quad \mathbf{V}_{J_{k,n}} \in \varphi_n, \forall n = 1, 2, \cdots, N. \tag{17}$$

In the above proposed per-cell product codebook based limited feedback scheme, the aggregate codeword $\bar{\mathbf{V}}\left(J_{k,1}, J_{k,2}, \cdots, J_{k,N}\right)$ can be thought as a codeword in the *product codebook* $\varphi_{Per} = \varphi_1 \otimes \varphi_2 \otimes \cdots \otimes \varphi_n$, i.e.,

$$\bar{\mathbf{V}}\left(J_{k,1}, J_{k,2}, \cdots, J_{k,N}\right) \in \varphi = \varphi_1 \otimes \varphi_2 \otimes \cdots \otimes \varphi_n, \tag{18}$$

where $\mathbf{V}_{J_{k,n}} \in \varphi_n$, $\forall n = 1, 2, \cdots, N$. The product codebook φ is important because it allows for a single codebook to be designed, and the real codebook that is used for CSI feedback is simply the Cartesian product of N single cell codebooks $\{\varphi_n\}_{n=1}^{N}$.

Remark 3 (Backward Compatibility): When N equals 1, i.e., the single-BS scenario, *Problem 1* will degenerate to the conventional feedback index generation problem.

C. Low-complexity Solution

Problem 1 belongs to the standard combinatorial search problem [24] and the optimal solution requires *exhaustive search* over the N per-cell product codebooks, which has exponential complexity w.r.t. the number of feedback-bits B_k. However, in the practical communication systems, a MS may not be able to support such complicated operations. In order to address this issue, we shall propose a low-complexity searching algorithm, which exploit the per-cell product codebook structure and decomposes the searching process over the N codebooks into the searching over N *sub-codebooks* with reduced size. For illustration purpose, we shall first give the definition of *sub-codebook*.

Definition 2 (Sub-codebook): A *sub-codebook* $\bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right)$, is defined as a collection of codewords in the original codebook φ_n, which lies in the neighborhood of δ_n of the quantization source $\mathbf{V}_{k,n}^{(w)}$. Mathematically, we have[7],

$$\bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right) \triangleq \left\{\mathbf{V} \,\middle|\, \mathbf{V} \in \varphi_n; d_c\left(\mathbf{V}, \mathbf{V}_{k,n}^{(w)}\right) < \delta_n\right\}. \tag{19}$$

Based on *Definition 2*, we can propose our low-complexity searching algorithm as well as complexity analysis in the following (see *Algorithm 1* below).

Remark 4 (Performance-Complexity Tradeoff of δ_n): In the above algorithm, the value of δ_n can be utilized to tradeoff the average quantization distortion performance and the computational complexity. In particular, when $\delta_1 = \delta_2 = \cdots = \delta_N = \sqrt{n_R}$, the above algorithm reduces to the traditional exhaustive search algorithm. As long as $\delta_n \leq \sqrt{n_R}$, $\forall n = 1, 2, \cdots, N$, then $\prod_{n=1}^{N}\left|\bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right)\right| \leq 2^{\sum_{n=1}^{B_{k,n}}} = 2^{B_k}$, which is the time complexity of the exhaustive search method for solving *Problem 1* with the original codebooks $\{\varphi_n\}_{n=1}^{N}$.

Algorithm 1 Low-complexity Indices Selection Algorithm (ISA)

- **Step 1: Find Localized Centroids**:
  At the $k^{th}(\forall k = 1, 2, \cdots, K)$ MS, the standard SVD operation is first applied to the CSI matrix $\mathbf{H}_{k,n}^{(w)}$, i.e.,

$$\begin{aligned}\mathbf{H}_{k,n}^{(w)} &= \mathbf{U}_{k,n}^{(w)}\left[\mathbf{S}_{k,n}^{(w)} \;\; \mathbf{0}_{n_R \times (n_T - n_R)}\right]\left[\mathbf{V}_{k,n}^{(w)} \;\; \widetilde{\mathbf{V}}_{k,n}^{(w)}\right] \\ &= \mathbf{U}_{k,n}^{(w)}\mathbf{S}_{k,n}^{(w)}\mathbf{V}_{k,n}^{(w)}.\end{aligned} \tag{20}$$

  Then, the $n_T \times (n_T - n_R)$ orthonormal basis $\mathbf{V}_{k,n}^{(w)}$ of the row space of the CSI matrix $\mathbf{H}_{k,n}^{(w)}$ is set as the centroid of the sub-codebook $\bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right)$. In this step, the standard SVD operation is with the time complexity of $\mathcal{O}\left(n_T n_R^2\right)$.
- **Step 2: Construct *Sub-codebooks***:
  After finding out the centroid $\mathbf{V}_{k,n}^{(w)}$ of the sub-codebook $\bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right)$, the $k^{th}(\forall k = 1, 2, \cdots, K)$ MS proceed to select all the codewords in codebook φ_n that are within the neighborhood δ_n of the centroid $\mathbf{V}_{k,n}^{(w)}$, to construct sub-codebook $\bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right)$. Specifically,

$$\begin{aligned}&\text{If } \mathbf{V} \in \varphi_n \text{ and } d_c\left(\mathbf{V},\; \mathbf{V}_{k,n}^{(w)}\right) < \delta_n, \\ &\quad \text{then } \mathbf{V} \in \bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right), \forall n = 1, 2, \cdots, N,\end{aligned}$$

  where the time complexity of this step is of the order $\mathcal{O}\left(\sum_{n=1}^{N} 2^{B_{k,n}}\right)$.
- **Step 3: Indices Selection with Sub-codebooks**:
  The $k^{th}(\forall k = 1, 2, \cdots, K)$ MS then searches the feedback indices within the restricted sub-codebooks $\left\{\bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right)\right\}_{n=1}$ to solve *Problem 1*. Specifically, the k^{th} MS tries to find out the feedback indices $\left\{J_{k,1}^*,\; J_{k,2}^*,\; \cdots,\; J_{k,N}^*\right\}$ through solving the following optimization problem with exhaustive search:

$$\begin{aligned}&\min_{J_{k,1}, J_{k,2}, \cdots, J_{k,N}} d_c(\bar{\mathbf{V}}\left(J_{k,1}, J_{k,2}, \cdots, J_{k,N}\right), \mathbf{V}_k^{(w)}) \\ &\text{subject to} \quad \mathbf{V}_{J_{k,n}} \in \bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right),\end{aligned} \tag{21}$$

  where the time complexity of this step is of the order of $\mathcal{O}\left(\prod_{n=1}^{N}\left|\bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right)\right|\right)$, with $\left|\bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right)\right|$ denoting the cardinality of the sub-codebook $\bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right)$, which depends on δ_n and $B_{k,n}$.

IV. Asymptotic Performance Analysis

In this section, we shall quantify the asymptotic performance of the proposed per-cell product codebook based limited feedback mechanism w.r.t. the system configurations (n_T, N, n_R, K) and the per-user feedback-bits B_k. In order to have tractable analysis so as to obtain design insights, we shall analyze the performance of the proposed limited feedback design using *random codebooks* [10], [18].

A. Asymptotic Optimality

We start with comparing the asymptotic performance of the proposed per-cell product codebook based limited feedback scheme and the joint-cell codebook approach. Denote φ_{Joint} and Φ_{Joint} as a random joint-cell codebook and the collection of all possible random joint-cell codebooks, respectively.

[7] In this paper, we use the *chordal distance* as the distance measure. Other distance metrics can also be applied here.

Similarly, denote φ_{Per} and Φ_{Per} as a random *product per-cell product codebook* and the collection of all possible random *product per-cell product codebooks*, respectively, where a random *product per-cell product codebook* is defined as the *Cartesian product* of N random per-cell product codebooks, i.e., $\varphi_{Per} = \varphi_1 \otimes \varphi_2 \otimes \cdots \otimes \varphi_N$. Let $\bar{D}_k(\Phi_{Joint})$ and $\bar{D}_k(\Phi_{Per})$ denote the *average quantization distortion* averaged over all possible random joint-cell codebooks and random per-cell product codebooks, respectively, which are defined as,

$$\bar{D}_k\big(\Phi_{Joint}\big) \triangleq \mathbb{E}\left\{\min_{\mathbf{V}\in\varphi_{Joint}} d_c^2\big(\mathbf{V},\ \mathbf{V}_k^{(w)}\big)\big|\mathbf{H}_k^{(w)};\varphi_{Joint}\in\Phi_{Joint}\right\}, \quad (22)$$

$$\bar{D}_k\big(\Phi_{Per}\big) \triangleq \mathbb{E}\left\{\min_{\mathbf{V}\in\varphi_{Per}} d_c^2\big(\mathbf{V},\ \mathbf{V}_k^{(w)}\big)\big|\mathbf{H}_k^{(w)};\varphi_{Joint}\in\Phi_{Per}\right\}. \quad (23)$$

In order to show the efficiency of the proposed per-cell product codebook based limited feedback scheme, we shall establish the *asymptotic optimality* of the proposed limited feedback design w.r.t. the joint-cell codebook approach and summarize the main results in the following Lemma.

Lemma 1 (Asymptotic Optimality): For sufficiently large n_T and finite N, we have:

(I) The $Nn_T \times n_R$ orthonormal basis $\mathbf{V}_k^{(w)}$ of the row-space of the $n_R \times Nn_T$ normalized CSI matrix $\mathbf{H}_k^{(w)}$ has the same structure as the aggregate-codeword defined in (15) almost surely (i.e., with probability 1);

(II) The proposed per-cell product codebook based limited feedback scheme and the joint-cell codebook approach achieve the same *average quantization distortion*, i.e.,

$$\bar{D}_k\left(\Phi_{Per}\right) = \bar{D}_k\left(\Phi_{Joint}\right),\ \forall\ k = 1, 2, \cdots, K. \quad (24)$$

Proof: Please refer to Appendix A for the proof. ■

By virtue of *Lemma 1*, we can derive the *average quantization distortion* associated with the random per-cell product codebooks, which is summarized in the following lemma.

Lemma 2 (Average Quantization Distortion): For sufficiently large B_k, n_T, and small n_R, the average quantization distortion associated with the random per-cell product codebooks is given by,

$$\bar{D}_k\left(\Phi_{per}\right) \approx n_R 2^{-\frac{B_k}{n_R(Nn_T - n_R)}}. \quad (25)$$

Proof: Please refer to Appendix B for the proof. ■

Remark 5 (Average Quantization Distortion): In *Lemma 2* the average quantization distortion is associated with quantizing the row-space of the normalized CSI matrix $\mathbf{H}_k^{(w)}$, which consists of $\mathbb{CN}(0,1)$ entries. As a result, the expression given in (25) is the same as equation (8) given in reference [10], which is in fact first proved in reference [18].

In the rest of this section, we shall derive the asymptotic performance of the proposed limited feedback design based on the above two lemmas, and study the effects of limited feedback and the advantage of macrodiversity provided by BS cooperation.

B. Effect of Limited Feedback

Within the framework of limit feedback study, the *throughput loss* due to CSI quantization is a common performance measure [8]–[11]. In this paper, we extend the concept into network MIMO configuration and define *per-user throughput loss* R_k^{Loss} as follows.

Definition 3 (Per-user Throughput Loss): The per-user throughput loss R_k^{Loss} (w.r.t. the k^{th} MS) is defined as the throughput gap between the global CSI[8] (GCSI) case and the proposed per-cell product codebook based limited feedback design, i.e.,

$$R_k^{Loss} = R_k^{CSIT} - R_k^{LF},\ \forall\ k = 1, 2, \cdots, K, \quad (26)$$

where CSIT is the abbreviation for Channel State Information at the Transmitter Side and LF is the abbreviation for Limited Feedback. Moreover,

$$R_k^{CSIT} = \mathbb{E}\left\{\log_2\det\left(\mathbf{I}_{n_R} + \frac{1}{\sigma^2}\mathbf{H}_k\mathbf{W}_k\mathbf{P}_k\mathbf{W}_k^\dagger\mathbf{H}_k^\dagger\right)\right\}, \quad (27)$$

$$R_k^{LF} = \mathbb{E}\Bigg\{\log_2\det\Bigg(\mathbf{I}_{n_R} + \Bigg(\sigma^2\mathbf{I}_{n_R} + \mathbf{H}_k\sum_{j=1,j\neq k}^{K}\widehat{\mathbf{W}}_j\mathbf{P}_j\widehat{\mathbf{W}}_j^\dagger\mathbf{H}_k^\dagger\Bigg)^{-1}\mathbf{H}_k\widehat{\mathbf{W}}_k\mathbf{P}_k\widehat{\mathbf{W}}_k^\dagger\mathbf{H}_k^\dagger\Bigg)\Bigg\}, \quad (28)$$

with $\mathbf{W}_k$ denoting the percoder for the k^{th} MS designed based on GCSI. Note that the residual CCI term in equation (28) is due to limited feedback effects, and the expectation operation is taken over Rayleigh fading and the lognormal shadowing effect, as well as the random per-cell product codebooks (for R_k^{LF} only). ■

The following theorem quantifies the asymptotic per-user throughput loss of the proposed limited feedback scheme w.r.t. the network configuration (n_T, N, n_R, K), the per-user feedback-bits B_k and the path loss geometry $\{g_{k,1}, g_{k,2}, \cdots, g_{k,N}\}$, which consists of all the path losses between the N cooperating BSs and the k^{th} MS, and $\{g_{k,n}\}$ has been normalized to the weakest path so that $g_{k,n} \geq 1$.

Theorem 1 (Asymptotic Per-user Throughput Loss): In the network MIMO system with the proposed *per-cell product codebook based limited feedback scheme*, the asymptotic per-user throughput loss is given by,

$$R_k^{Loss} = \mathcal{O}\left(n_R\log_2\left(2^{-\frac{B_k}{n_R(Nn_T - n_R)}}\rho g_k^{sum}\right)\right), \quad (29)$$

where $\rho = \frac{P_{max}}{\sigma^2}$ is termed as *system SNR*; g_k^{sum} is defined to be $\sum_{n=1}^{N} g_{k,n}$.

Proof: Please refer to Appendix C for the proof. ■

As direct consequences of *Theorem 1*, we have the following corollaries.

Corollary 1 (Scaling Law for the Noise-Limited Regime): For the per-cell product codebook based feedback scheme, the per-user feedback-bits B_k required to bound the per-user throughput loss within a constant ε shall scale according to the following expression,

$$B_k \approx n_R(Nn_T - n_R)\log_2\left(\rho g_k^{sum}\right) - c(\varepsilon), \quad (30)$$

[8] In this paper, global CSI means that all the cooperating BSs has perfect CSI knowledge of the whole network.

where $c(\varepsilon) = n_R(Nn_T - n_R)\log_2\left(2^{\frac{\varepsilon}{n_R}} - 1\right)$.

Proof: Setting right hand side (RHS) of equation (49) in *Appendix C* to ε, and solving for B_k will result in equation (30) directly. ■

Corollary 2 (Scaling Law for the Interference-Limited Reg For the proposed per-cell product codebook limited feedback scheme, if per-user feedback-bits (i.e., B_k) does not scale with system SNR ρ, then for sufficiently large P_{max}, the per-user throughput R_k^{LF} tends to a constant and scales according to,

$$R_k^{LF} = \mathcal{O}\left(\frac{n_R B_k \ln 2}{(Nn_T - n_R)\, Nn_T}\right), \ \forall k = 1, 2, \cdots, K. \tag{31}$$

Proof: Please refer to Appendix D for the proof. ■

Remark 6 (Effects of Heterogeneous Path Losses): Note that, the results given in *Theorem 1* and *Corollary 1* above are similar to those results stated in Theorem 1 and Theorem 2 of reference [10]. The major difference is the path loss effect term g_k^{sum}, which results from the different path losses from the N cooperating BSs to the k^{th} MS.

Remark 7 (Scaling Laws for the Proposed Limited Feedback Design): In the noise-limited regime, the minimum number of feedback-bits B_k required to maintain a bounded per-user throughput loss, shall scale w.r.t. the number of cooperating BSs according to,

$$B_k = \mathcal{O}\bigg(Nn_T n_R \log_2\left(\rho g_k^{sum}\right)\bigg). \tag{32}$$

Moreover, the residual CCI term in equation (28) is negligible for the noise-limited case. Following the proof of *Theorem 1*, we can show that in the noise-limited regime, the achievable per-user throughput of the proposed limited feedback scheme scales as,

$$R_k^{LF} = \mathcal{O}\left(n_R \log_2\left(\frac{n_R \rho g_k^{sum}}{Nn_T}\right)\right). \tag{33}$$

On the other hand, in the interference-limited regime, the achievable per-user throughput of the proposed per-cell product codebook based limited feedback scheme shall scale as,

$$R_k^{LF} = \mathcal{O}\left(\frac{n_R B_k}{(Nn_T)^2}\right). \tag{34}$$

V. Numerical Results and Discussions

In this section, we shall study the performance of the proposed per-cell product codebook limited feedback scheme and verify the analytical results via simulations. We shall first compare the proposed per-cell product codebook limited feedback scheme with several baseline schemes. *Baseline 1: joint-cell codebook approach*; *Baseline 2 and 3*: Givens rotation approach with different number of feedback-bits. In the Givens rotation approach, the two Givens parameters of a Givens matrix are quantized with a two-dimensional vector quantizer [19]–[21]. Then, we proceed to study the performance-complexity tradeoff of the proposed low-complexity feedback indices selection algorithm, as well as the analytical results in Section IV. In the simulations, two-dimensional hexagonal cellular model is considered, with a cell-radius of 300 meters and the carrier frequency is set to be 2 GHz. The path loss model specified in [25] is used, i.e., $PL(dB) = 130.19 + 37.6\log_{10}(d(km))$, with 8 dB lognormal shadowing effects. Users are assumed to be uniformly distributed within the cooperating cells. We use *interference-free SNR* to represent the receiving SNR at the cell edge of a single-cell single-MS scenario.

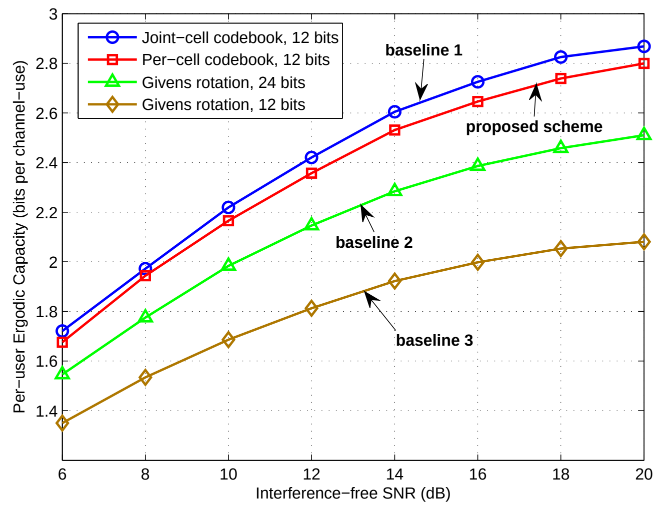

Fig. 2. Performance comparison of the proposed per-cell product codebook based limited feedback design with joint-cell codebook and Givens rotation approaches, in a network MIMO system $(n_T, N, n_R, K) = (4, 3, 2, 6)$. In the proposed scheme, baseline 1 and baseline 3, we adopt 4 bits per BS for CSI feedback; while in baseline 2, we adopt 8 bits per BS for CSI feedback. While the proposed per-cell product codebook limited feedback scheme can achieve $95\% \sim 97\%$ of the performance of joint-cell codebook approach, it performs much better than the Givens rotation approach.

A. Performance of the Proposed Limited Feedback Design

Fig. 2 illustrates the per-user average throughput versus interference-free SNR. The simulation results show that, in the practical settings, the proposed per-cell product codebook based limited feedback scheme can achieve $95\% \sim 97\%$ the performance of the joint-cell codebook approach. Moreover, the proposed scheme achieves much better throughput compared with *Baseline 2* (Givens rotation approach with doubled number of bits for limited feedback) and *Baseline 3* (Givens rotation approach). This is because the Givens rotation approach has quite low feedback efficiency due to two-dimensional vector quantization, compared with matrix quantization in the codebook-based approach.

B. Performance-complexity Tradeoff of the Low-complexity ISA

In this section, we study the performance-complexity tradeoff of the low-complexity ISA. Here, *performance* is measured in terms of *per-user ergodic capacity*, and *complexity* is measured in terms of *the number of codewords* that are searched for generating the codewords indices. As stated in section III-C, the parameters δ_n $(n = 1, 2, \cdots, N)$ determine the tradeoff between performance and complexity of *algorithm 1*. Fig. 3 shows the performance-complexity tradeoff with different choices of δ_n, where we set δ_n to the same value for all n. The complexity numbers shown in the figure denote

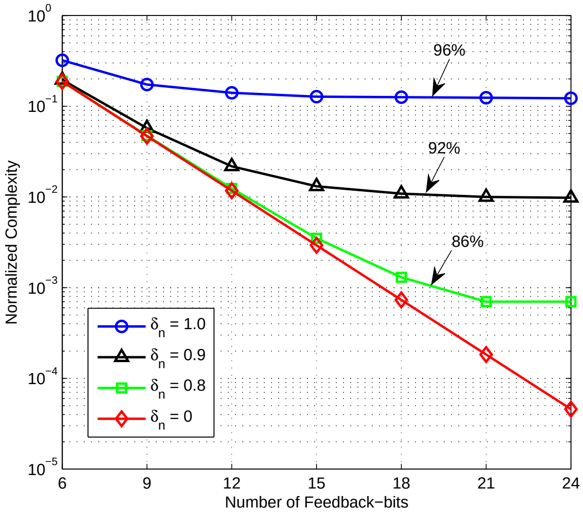

Fig. 3. Performance-complexity tradeoff of the proposed low-complexity feedback indices selection algorithm (*algorithm 1*), with different choices of δ_n for a network MIMO system $(n_T, N, n_R, K) = (4, 3, 2, 6)$. The complexity numbers shown in the figure are relative complex defined in equation (35). Setting $\delta_n = 1, 0.9, 0.8$ can achieve 96%, 92%, 86% the performance of exhaustive search, with 10%, 1%, 0.1% the complexity of exhaustive search, respectively. Larger δ_n gives better performance at the cost of higher complexity, and vice versa. The curve with $\delta_n = 0$ is for reference.

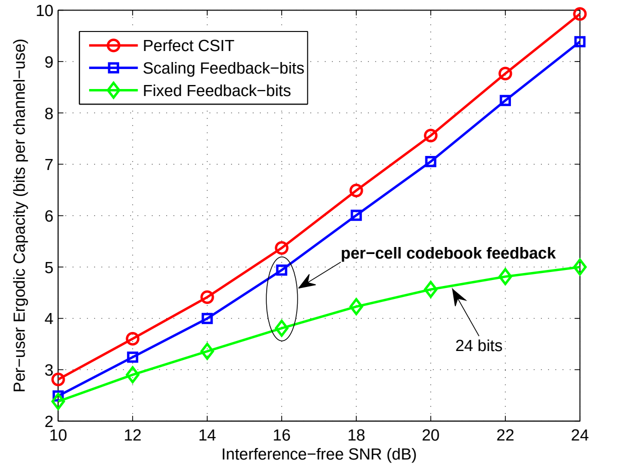

Fig. 4. Per-user ergodic capacity versus interference-free SNR, for perfect CSIT case, per-cell product codebook feedback with scaling feedback-bits and fixed feedback-bits, with a system configuration $(n_T, N, n_R, K) = (8, 3, 2, 12)$. For the scaling feedback-bits case, the per-user feedback-bits scale according to equation (30), and we set $\varepsilon = 1$. Specifically, the feedback-bits used in the scaling feedback-bits case are $[24\ 30\ 36\ 42\ 48\ 57\ 63\ 72]$, corresponding to the 8 SNR values respectively. A constant gap of about 0.5 (bits per channel use) between the scaling feedback and the perfect CSIT case is observed. For the fixed feedback-bits case, the system is interference-limited.

the relative complexity with respect to the exhaustive search method with original codebooks, i.e.,

$$\begin{aligned}&\text{Relative complexity}\\ &\triangleq \frac{\text{No. of codewords searched with } \textit{Algorithm 1}}{\text{No. of codewords searched with exhaustive search}}\\ &= \frac{\prod_{n=1}^{N} \left| \bar{\varphi}\left(\mathbf{V}_{k,n}^{(w)}, \delta_n\right)\right|}{2^{B_k}}. \end{aligned} \tag{35}$$

As shown in the figure, with $\delta_n = 0.9$, the low-complexity ISA can achieve 92% the performance of the exhaustive search approach, with only 1% complexity of the exhaustive search. With larger δ_n, *algorithm 1* can achieve better performance at the cost of higher computational complexity, and vice versa.

C. Verification of the Analytical Expressions

In this section, we shall compare the analytical results stated in *Corollary 1* and *Corollary 2* with numerical results, and demonstrate the validity of those analytical studies. Fig. 4 shows the simulation results for GCSI case, per-cell product codebook limited feedback with scaling feedback-bits and fixed feedback-bits. For the per-cell product codebook based limited feedback with scaling feedback-bits, we set $\varepsilon = 1$ and let the feedback-bits scale according to equation (30). Compared with GCSI case, about 0.5 (bits per channel use) throughput loss is achieved with scaling-feedback.

Fig. 5 shows the performance of the proposed per-cell product codebook based limited feedback scheme in interference limited regime, with different number of feedback-bits per BS (i.e., $\frac{B_k}{N}$). We can see that the per-user ergodic capacity scales linearly w.r.t. feedback-bits per BS under all system configurations (n_T, N, n_R, K). The simulation results shown in Fig. 5 match the analytical results stated in *Corollary 2*.

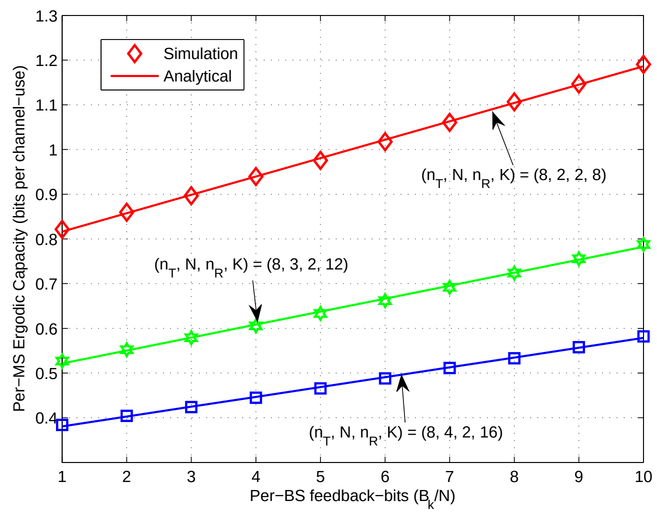

Fig. 5. Per-user throughput in the interference-limited regime with different system configurations (n_T, N, n_R, K). The y-axis is the per-user ergodic capacity (in bits per channel use), and the x-axis is the per-BS feedback-bits. It is observed that the per-user throughput scale linearly with the feedback-bits, as stated in *Corollary 2*. The simulation results match the analytical results stated in *Corollary 2*.

VI. Conclusions

In this paper, we have proposed a scalable per-cell product codebook based limited feedback framework for network MIMO systems, along with a low-complexity feedback indices selection algorithm. We have shown that the proposed limited feedback design can asymptotically achieve the same performance as the joint-cell codebook approach. When the number of per-user feedback-bits scales as $\mathcal{O}\big(N n_T n_R \log_2(\rho g_k^{sum})\big)$, the proposed scheme operates in a noise-limited regime with a per-user throughput scaling as $\mathcal{O}\left(n_R \log_2\left(\frac{n_R \rho g_k^{sum}}{N n_T}\right)\right)$. On

the other hand, when the number of per-user feedback-bits does not scale with system SNR, the system operates in an interference-limited regime with a per-user throughput scaling as $\mathcal{O}\left(\frac{n_R B_k}{(Nn_T)^2}\right)$. The numerical results show that the proposed scheme can achieve similar performance as the joint-cell codebook approach and performs much better than the Givens rotation approach in practical settings. One interesting direction for further study is to consider adaptive feedback-bits allocation based on users path loss geometry.

Appendix A
Proof of Lemma 1

For ease of elaboration, we first introduce the following intermediate lemma.

Lemma 3: Consider a random vector $\mathbf{h} \in \mathbb{C}^{Nn_T}$ with i.i.d. $\mathbb{CN}(0,\sigma^2)$ entries, and let $\tilde{\mathbf{h}} = \frac{\mathbf{h}}{\|\mathbf{h}\|}$ denote the direction of $\mathbf{h}$. Partitioning $\tilde{\mathbf{h}}$ into N *sub-vectors*, i.e., $\tilde{\mathbf{h}} = \left[\tilde{\mathbf{h}}_1^\dagger \ \tilde{\mathbf{h}}_2^\dagger \ \cdots \ \tilde{\mathbf{h}}_N^\dagger\right]^\dagger$, where $\tilde{\mathbf{h}}_n \in \mathbb{C}^{n_T}$, $\forall n = 1,2,\cdots,N$, we then have,

$$\Pr\left\{\lim_{n_T\to\infty}\left\{\tilde{\mathbf{h}}_n^\dagger\tilde{\mathbf{h}}_n\right\} = \frac{1}{N}\right\} = 1,\ \forall\ n = 1,2,\cdots,N. \quad (36)$$

Proof: Denote $\tilde{h}_i$ as the i^{th} element of $\tilde{\mathbf{h}}$. Since $\mathbf{h}$ is a random vector with i.i.d. $\mathbb{CN}(0,\sigma^2)$ entries, all $\tilde{h}_i$ $(i = 1,2,\cdots,Nn_T)$ are identically distributed and satisfy,

$$\mathit{Var}\left\{\tilde{h}_i\tilde{h}_i^\dagger\right\} = \frac{Nn_T - 1}{(Nn_T)^2(Nn_T+1)},\ \forall\ i = 1,2,\cdots,Nn_T, \quad (37)$$

where $\mathit{Var}\left\{\tilde{h}_i\tilde{h}_i^\dagger\right\}$ denotes the variance of $\tilde{h}_i\tilde{h}_i^\dagger$. Moreover, it is straightforward to get that $\mathbb{E}\left\{\tilde{\mathbf{h}}_n^\dagger\tilde{\mathbf{h}}_n\right\} = \frac{1}{N}$, and $\mathit{Var}\left\{\tilde{\mathbf{h}}_n^\dagger\tilde{\mathbf{h}}_n\right\} = \frac{N-1}{N^2(Nn_T+1)}$, $\forall\ n = 1,2,\cdots,N$.

Define $\psi_t = \left\{\left|\tilde{\mathbf{h}}_n^\dagger\tilde{\mathbf{h}}_n - \frac{1}{N}\right| > \epsilon\right\}$, where ϵ denotes any given positive number. Using Chebyshev's inequality, we get,

$$\Pr\{\psi_t\} = \Pr\left\{\left|\tilde{\mathbf{h}}_n^\dagger\tilde{\mathbf{h}}_n - \frac{1}{N}\right| > \epsilon\right\} \le \frac{N-1}{N^2(Nt+1)\epsilon^2}, \quad (38)$$

which implies that [26, page 37],

$$\Pr\left\{\bigcup_{t\ge n_T}\psi_t\right\} \le \sum_{t\ge n_T}\Pr\{\psi_t\} \to 0 \quad \text{as} \quad n_T\to\infty, \quad (39)$$

which proves *Lemma 3* [26, pape 34-35]. ■

We next proceed to prove the first statement of *Lemma 1*. We first partition $\mathbf{V}_k^{(w)} \in \mathbb{C}^{Nn_T\times n_R}$ into N sub-matrices, i.e.,

$$\mathbf{V}_k^{(w)} = \left[\left[\mathbf{V}_{k,1}^{(w)}\right]^\dagger \ \left[\mathbf{V}_{k,2}^{(w)}\right]^\dagger \ \cdots \ \left[\mathbf{V}_{k,N}^{(w)}\right]^\dagger\right]^\dagger, \quad (40)$$

where $\mathbf{V}_{k,n}^{(w)} \in \mathbb{C}^{n_T\times n_R}$, $\forall\ n = 1,2,\cdots,N$. Then, as a direct consequence of *Lemma 3*,

$$\Pr\left\{\lim_{n_T\to\infty}\left\{\left[\mathbf{V}_{k,n}^{(w)}\right]^\dagger\mathbf{V}_{k,n}^{(w)}\right\} = \frac{1}{N}\mathbf{I}_{n_R}\right\} = 1, \quad (41)$$

Equation (41) suggests that when n_T is sufficiently large, the orthonormal basis $\mathbf{V}_k^{(w)}$ shall have the same structure as the *aggregate-codeword* defined in (15) almost surely (i.e., with probability 1), which proves the first statement of *Lemma 3*.

Note that, the first statement of *Lemma 1* in turn suggests that the codewords in the joint-cell codebook shall have the same structure as the *aggregate-codeword* defined in (15) almost surely for sufficiently large n_T and finite N. As a result, the second statement of *Lemma 1* can be derived from the definition of average distortion associated with joint-cell codebooks and per-cell product codebooks given in equation (22) and (23), respectively. Therefore, the expected distortion (i.e., average distortion) associated with the random per-cell product codebooks shall be the same as the expected distortion (i.e., average distortion) with random joint-cell codebooks.

Appendix B
Proof of Lemma 2

By virtue of *Lemma 1*, for large B_k, the average quantization distortion associated with the random per-cell product codebooks can be approximated as [18],

$$\bar{D}_k(\Phi_{per}) \approx \frac{\Gamma\left(\frac{1}{\alpha}\right)}{\alpha}\beta^{-\frac{1}{\alpha}}2^{-\frac{B_k}{\alpha}} + o(1) \quad (42)$$

where $\alpha = n_R(Nn_T - n_R)$; $\beta = \frac{1}{\alpha!}\prod_{i=1}^{n_R}\frac{(Nn_T-i)!}{(n_R-i)!}$; $\Gamma(x)$ denotes the Gamma function; the $o(1)$ term can be ignored when B_k is large or n_R is small [18]. Since $\lim_{\alpha\to\infty}\frac{\Gamma\left(\frac{1}{\alpha}\right)}{\alpha} = 1$, for large α (which is true in network MIMO system), we have $\frac{\Gamma\left(\frac{1}{\alpha}\right)}{\alpha} \approx 1$.

Substituting Stirling's approximation for factorial, we get $\beta \approx \frac{(Nn_T)^{\frac{n_R-1}{2}}}{(n_R)^{Nn_Rn_T}}$ and,

$$\beta^{-\frac{1}{\alpha}} \approx \frac{n_R}{(Nn_T)^{\frac{n_R-1}{2Nn_Rn_T}}} \overset{(a)}{\approx} n_R, \quad (43)$$

where (a) is because of that $\lim_{n_T\to\infty}(Nn_T)^{\frac{n_R-1}{2Nn_Rn_T}} = 1$ and Nn_T is usually very large for network MIMO system. Therefore, the approximation of the average quantization distortion associated with the random per-cell product codebooks $\bar{D}_k(\Phi_{per})$ can be further simplified as $\bar{D}_k(\Phi_{per}) \approx n_R 2^{-\frac{B_k}{n_R(Nn_T-n_R)}}$.

Appendix C
Proof of Theorem 1

Here is the proof of *Theorem 1*.

$$\begin{aligned}
R_k^{Loss} &\overset{(a)}{\le} \mathbb{E}\Big\{\log_2\det\Big(\mathbf{I}_{n_R} \\
&\quad + \frac{1}{\sigma^2}\mathbf{H}_k\sum_{j=1,j\ne k}^{K}\widehat{\mathbf{W}}_j\mathbf{P}_j\widehat{\mathbf{W}}_j^\dagger\mathbf{H}_k^\dagger\Big)\Big\} \\
&\overset{(b)}{=} \mathbb{E}\Big\{\log_2\det\Big(\mathbf{I}_{n_R} + \frac{1}{\sigma^2}\big[\mathbf{V}_k^{(w)}\big]^\dagger\mathbf{G}_k \\
&\quad \sum_{j=1,j\ne k}^{K}\widehat{\mathbf{W}}_j\mathbf{P}_j\widehat{\mathbf{W}}_j^\dagger\mathbf{G}_k\mathbf{V}_k^{(w)}\left[\mathbf{S}_k^{(w)}\right]^2\Big)\Big\} \\
&\overset{(c)}{\le} \log_2\det\Big(\mathbf{I}_{n_R} + \frac{1}{\sigma^2}\mathbb{E}\Big\{\big[\mathbf{V}_k^{(w)}\big]^\dagger\mathbf{G}_k
\end{aligned}$$

$$\sum_{j=1,j\neq k}^{K}\widehat{\mathbf{W}}_j\mathbf{P}_j\widehat{\mathbf{W}}_j^{\dagger}\mathbf{G}_k\mathbf{V}_k^{(w)}\left[\mathbf{S}_k^{(w)}\right]^2\Big\}),$$

where (a) and (c) are obtained following the approaches in [10]; (b) follows by substituting equation (10) for $\mathbf{H}_k^{(w)}$.

Let $\mathbf{F}_k=\mathbb{E}\Big\{\left[\mathbf{V}_k^{(w)}\right]^{\dagger}\mathbf{G}_k\sum_{j=1,j\neq k}^{K}\widehat{\mathbf{W}}_j\mathbf{P}_j\widehat{\mathbf{W}}_j^{\dagger}\mathbf{G}_k\mathbf{V}_k^{(w)}\left[\mathbf{S}_k^{(w)}\right]^2\Big\}$, and note that $\mathbf{V}_k^{(w)}$, $\mathbf{G}_k$, $\widehat{\mathbf{W}}_j$ and $\left[\mathbf{S}_k^{(w)}\right]^2$ are mutually independent, the expectation can be carried out step by step.

- Step 1: substituting the decomposition of $\mathbf{V}_k^{(w)}$, i.e., $\mathbf{V}_k^{(w)}=\widehat{\mathbf{V}}_k^{(w)}\mathbf{X}_k\mathbf{Y}_k+\widetilde{\mathbf{V}}_k^{(w)}\mathbf{Z}_k$ (see *Lemma 1* of [10]).

$$\begin{aligned}\mathbf{F}_k= &\ \mathbb{E}\left\{\left[\mathbf{V}_k^{(w)}\right]^{\dagger}\mathbf{G}_k\sum_{j=1,j\neq k}^{K}\widehat{\mathbf{W}}_j\mathbf{P}_j\widehat{\mathbf{W}}_j^{\dagger}\mathbf{G}_k\mathbf{V}_k^{(w)}\right\}\\ &\ \mathbb{E}^{(1)}\left\{\left[\mathbf{S}_k^{(w)}\right]^2\right\}\\ \overset{(d)}{=} &\ Nn_T\mathbb{E}\left\{\mathbf{Z}_k^{\dagger}\left[\widetilde{\mathbf{V}}_k^{(w)}\right]^{\dagger}\mathbf{G}_k\right.\\ &\left.\sum_{j=1,j\neq k}^{K}\widehat{\mathbf{W}}_j\mathbf{P}_j\widehat{\mathbf{W}}_j^{\dagger}\mathbf{G}_k\widetilde{\mathbf{V}}_k^{(w)}\mathbf{Z}_k\right\},\end{aligned}\tag{44}$$

  where (d) is because of that $\mathbb{E}^{(1)}\left\{\left[\mathbf{S}_k^{(w)}\right]^2\right\}=Nn_T\mathbf{I}_{n_R}$ (see Appendix B of [10]) and we have used the LF-BD conditions (2).
- Step 2: calculating the expectation of $\left(\sum_{j=1,j\neq k}^{K}\widehat{\mathbf{W}}_j\mathbf{P}_j\widehat{\mathbf{W}}_j^{\dagger}\right)$. Let

$$\mathbf{Q}_k=\mathbb{E}^{(2)}\left\{\sum_{j=1,j\neq k}^{K}\widehat{\mathbf{W}}_j\mathbf{P}_j\widehat{\mathbf{W}}_j^{\dagger}\right\},\tag{45}$$

  where $\mathbb{E}^{(2)}$ denotes expectation taken over the distribution of $\widehat{\mathbf{W}}_j$. When the number of active users is large and the users are randomly distributed, we can safely conclude that $\mathbf{Q}_k\approx\frac{n_R(K-1)}{Nn_T}p\mathbf{I}_{Nn_T}$, with $p=\frac{NP_{max}}{Kn_R}$.
- Step 3: calculating expectation over $\widetilde{\mathbf{V}}_k^{(w)}$.

$$\mathbb{E}^{(3)}\left\{\left[\widetilde{\mathbf{V}}_k^{(w)}\right]^{\dagger}\mathbf{G}_k\mathbf{Q}_k\mathbf{G}_k\widetilde{\mathbf{V}}_k^{(w)}\right\}=\frac{pn_R\gamma_k\left(K-1\right)}{\left(Nn_T\right)^2}\mathbf{I}_{n_R}\tag{46}$$

  where $\mathbb{E}^{(3)}$ denotes expectation taken over the distribution of $\widetilde{\mathbf{V}}_k^{(w)}$ (isotropic distribution), and $\gamma_k=n_T\sum_{n=1}^{N}g_{k,n}s_{k,n}$.
- Step 4: calculating expectation over lognormal-shadowing. Let

$$\tilde{\gamma}_k=\mathbb{E}^{(4)}\left\{\gamma_k\right\}=n_Tg_k^{sum},\tag{47}$$

  where $\mathbb{E}^{(4)}$ denotes expectation taken over lognormal-shadowing.
- Step 5: calculating expectation over quantization error $\mathbf{Z}_k$. We have,

$$\frac{n_R\tilde{\gamma}_k}{\left(Nn_T\right)^2}p\mathbb{E}^{(5)}\left\{\mathbf{Z}_k^{\dagger}\mathbf{Z}_k\right\}\overset{(e)}{=}\frac{p\tilde{\gamma}_k\bar{D}\left(K-1\right)}{\left(Nn_T\right)^2}\mathbf{I}_{n_R},\tag{48}$$

  where $\mathbb{E}^{(5)}$ denotes expectation taken over the distribution of $\mathbf{Z}_k$, and (e) is given in Appendix B of [10] with $\bar{D}=n_R2^{-\frac{B_k}{n_R(Nn_T-n_R)}}$.
- Step 6: Finally, substituting equation (48) into (44), we get,

$$\mathbf{F}_k=\frac{p\tilde{\gamma}_k}{Nn_T}\bar{D}\mathbf{I}_{n_R}=\frac{p\bar{D}\left(K-1\right)g_k^{sum}}{N}\mathbf{I}_{n_R}.\tag{49}$$

Therefore, the asymptotic per-user throughput loss due to limited feedback is given by,

$$R_k^{Loss}=\mathcal{O}\left(n_R\log_2\left(2^{-\frac{B_k}{n_R(Nn_T-n_R)}}\rho g_k^{sum}\right)\right).\tag{50}$$

Appendix D
Proof of Corollary 2

The proof of *Corollary 2* can be summarized as follows.

$$\begin{aligned}R_k^{IFL}\overset{(a)}{\approx}&\ \mathbb{E}\left\{\log_2\det\left(\mathbf{H}_k\sum_{j=1}^{K}\widehat{\mathbf{W}}_j\widehat{\mathbf{W}}_j^{\dagger}\mathbf{H}_k^{\dagger}\right)\right\}\\ &-\ \mathbb{E}\left\{\log_2\det\left(\mathbf{H}_k\sum_{j=1,j\neq k}^{K}\widehat{\mathbf{W}}_j\widehat{\mathbf{W}}_j^{\dagger}\mathbf{H}_k^{\dagger}\right)\right\}\\ \overset{(b)}{\approx}&\ \log_2\det\left(\mathbb{E}\left\{\mathbf{H}_k\sum_{j=1}^{K}\widehat{\mathbf{W}}_j\widehat{\mathbf{W}}_j^{\dagger}\mathbf{H}_k^{\dagger}\right\}\right)\\ &-\ \log_2\det\left(\mathbb{E}\left\{\mathbf{H}_k\sum_{j=1,j\neq k}^{K}\widehat{\mathbf{W}}_j\widehat{\mathbf{W}}_j^{\dagger}\mathbf{H}_k^{\dagger}\right\}\right)\\ \overset{(c)}{=}&\ \mathcal{O}\left(n_R\log_2\left(1+\frac{2^{\frac{B_k}{n_R(Nn_T-n_R)}}}{K-1}\right)\right)\\ \overset{(d)}{=}&\ \mathcal{O}\left(\frac{B_k\ln 2}{\left(Nn_T-n_R\right)\left(K-1\right)}\right),\end{aligned}\tag{51}$$

where (a) is because the noise term is negligible compared with the CCI term; (b) holds in the *orderwise* sense; (c) follows the same arguments as in the proof of *Theorem 1*; (d) is because $\frac{B_k}{n_R(Nn_T-n_R)}$ is quite small and $2^{\frac{B_k}{n_R(Nn_T-n_R)}}\approx 1+\frac{B_k\ln 2}{n_R(Nn_T-n_R)}$.

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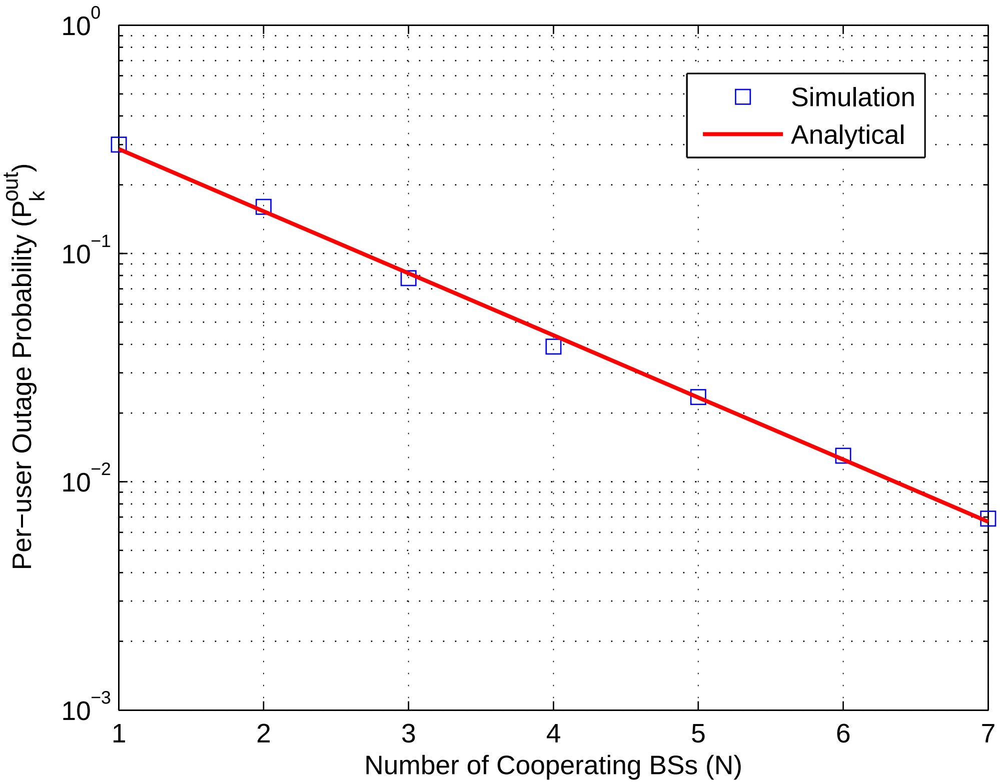

Simulation
Analytical
Per−user Outage Probability (P_k^{out})
Number of Cooperating BSs (N)
10^0
10^{-1}
10^{-2}
10^{-3}
1
2
3
4
5
6
7